believing that such are far more permanent and therefore more racial than the minor variations which have engaged the attention of others. His arguments are drawn from a conscientious study of ample series from various quarters of the globe, and though some of his refinements may not be sufficiently established, the general principles he advocates merit the careful consideration of cranial specialists, as containing some new and certainly correct observations. A short prefatory note by myself introduces the author to the American public.

THE ARYAN CRADLE-LAND.

IF anybody thinks that the question whether the primitive Aryan horde lived in Europe or Asia has been settled, he is mistaken. Two publications of late date show that the defenders of the old theory of their central Asian origin are nowise lacking in vigorous argument.

Prof. August Boltz, of Darmstadt, in a pamplet *Das Vedavolk in seinen Gesamtverhältnissen*, has worked out the problem of the origin and earliest migrations of the Aryans quite to his own satisfaction. He adds two maps, on which the reader can trace very clearly how they began in the great Tarim basin and about Lob Nor, and journeyed westward across the Pamir plateau, on the western slope of which they diverged, the Celtic stem wandering northwest into Europe north of the Black Sea; the Greek, Latin, Etruscan and Slavic branches by way of the Hellespont and the islands; the Iranian group remaining in Persia, while the Veda-folk or Indo-Aryans, ascended the mighty passes of the Hindu Kusch and Karakorum ranges to reach the fertile valleys to the south. These are pretty plans, but we look in vain for a substantial support to them.

Turning to Europe, M. De Nadaillac's admirable summary of the results of the investigations in the lake-dwelling of that continent (in a contribution to the *Revue des Questions Scientifiques* for October last, entitled *Les Populations Lacustres de l'Europe*) lifts the veil as far as at present possible on European culture in neolithic times—those times when the Aryan stock began its wide wanderings. The writer inclines to their Asian origin; but with his customary frankness he acknowledges that nowhere in the debris of these ancient dwellings has there a single positive sign of Asiatic art been discovered, nor any relic such as we might suppose even a savage tribe would carry from its pristine home. Until down to a late period of prehistoric time, European culture seems to have been indigenous. For a clear and accurate summary of what it was among the lake-dwellers, the student would do well to peruse the article referred to.

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TCHÉBYCHEV.*

OF Russian mathematicians, second only to Lobachévsky should be ranked Pafnutij Lvovitsch Tchébychev.

Born in Russia in 1821 and formerly professor at the University at St. Petersburg, he reached deservedly the very highest scientific honors, being privy councillor, the representative of applied mathematics in the Imperial Academy of St. Petersburg, in 1860 made member of the famous Section I.–Géométrie, of the French Académie des Sciences, and afterward *Associé étranger*, the highest honor attainable by a foreigner.

His best known work is the justly celebrated *Mémoire sur les nombres premiers*, Académie Impériale de Saint Pétersbourg, (1850), where he established the existence of limits within which the sum of the logarithms of the primes inferior to a given number must be comprised. This memoir is given in *Liouville's Journal*, 1852, pp. 366–390.

* Deceased December 8, 1894.

Sylvester afterward contracted Tchébychev's limits; but the original paper remains highly remarkable, especially as it depends on very elementary considerations.

In this respect it is in striking contrast to the equally marvelous paper of the lamented Riemann, *Ueber die Anzahl der Primzahlen unter einer Gegegebenen Grösse* presented to the Berlin *Academia* in 1859. Tchébychev had in 1848 presented a paper with this very title to the St. Petersburg Academie; *Sur la totalité des nombres premiers inferieurs á une limite doninée.* (Given in Liouville's Journal, 1852, pp. 341–365.)

Riemann speaks of the interest long bestowed on this subject by Gauss and Dirichlet, but makes no mention of Tchébychev. However, Sylvester speaks of 'his usual success in overcoming difficulties insuperable to the rest of the world.'

But though best known for his work in the most abstract part of mathematics, in reality Tchébychev was of an eminently practical turn of mind.

Thus it was his work, *Theorie des mechanismes connus sous le nom de parallélogrammes* (Mémoirs des savants étrangers, Tom. VII.), which led him to the elaborate dissertation *Sur les questions de minima qui se rattachent a la représentation approximate des fonctions,* 91 quarto pages in Mémoirs de l' Académie Impériale des Sciences de Saint Pétersbourg, 1858. While the variable x remains in the vicinity of one same value we can represent with the greatest possible approximation any function $f(x)$, of given form, by the principles of the differential calculus. But this is not the case if the variable x is only required to remain within limits more or less extended. The essentially different methods demanded by this case, which is just the one met in practice, are developed in this memoir.

The same line of thought led to his connection with a subject which has since found a place even in elementary text-books, namely rectilineal motion by linkage.

He invented a three-bar linkage, which is called Tchébychev's parallel motion, and gives an extraordinarily close approximation to exact rectilineal motion; so much so that in a piece of apparatus exhibited by him in the London Loan Collection of Scientific Apparatus, a plane supported on a combination of two of his parallel motion linkages seemed to have a strictly horizontal movement, though its variation was double that of the tracer in the simple parallel motion.

Tchébychev long occupied himself with attempting to solve the problem of producing exact rectilineal motion by linkage, until he became convinced that it was impossible and even strove long to find a proof of that impossibility. What must have been his astonishment then, when a freshman student of his own class, named Lipkin, showed him the long sought conversion of circular into straight motion. Tchébychev brought Lipkin's name before the Russian government, and secured for him a substantial reward for his supposed original discovery.

And perhaps it was independent, but it had been found several years previously by a French lieutenant of engineers, Peaucellier, and first published by him in the form of a question in the *Annales de Mathematique* in 1864. When Tchébychev was on a visit to London, Sylvester inquired after the progress of his proof of the impossibility of exact parallel motion, when the Russian announced its double discovery and made a drawing of the cell and mounting. This Sylvester happened to show to Manuel Garcia, inventor of the laryngoscope, and the next day received from him a model constructed of pieces of wood fastened with nails as pivots, which, rough as it was, worked perfectly. Sylvester exhibited this to the Philosophical Club of the Royal So-

ciety and in the Athenæum Club, where it delighted Sir Wm. Thomson, now Lord Kelvin, and led to the extraordinary lecture *On Recent Discoveries in Mechanical Conversion of Motion,* delivered by Sylvester before the Royal Institution on January 23, 1874. This in turn led to Kempe's remarkable development of the subject, and to Hart's discovery of a five-bar linkage which does the same work as Peaucellier's of seven.

Henceforth Peaucellier's Cell and Hart's Contraparallelogram will take their place in our text-books of geometry, and straight lines can be drawn without begging the question by assuming first a straight edge or ruler as does Euclid.

Thus Kempe's charming book, '*How to Draw a Straight Line,*' is a direct outcome of Tchébychev's sketch for Sylvester. As might perhaps have been expected, the immortal Lobachévsky found in his compatriot a devoted admirer. Not only was Tchébychev an active member of the committee of the Lobachévsky fund, but he took the deepest interest in all connected with the spread of the profound ideas typified in the non-Euclidean geometry. Knowing this, Vasiliev in his last letter asked that a copy of my translation of his address on Lobachévsky be forwarded to the great man. His active participation in scientific assemblies is also worthy of note; for example, at the 'Congrès de l' association française pour l'avancement des sciences, à Lyon,' he read two interesting papers, *Sur les valeurs limites des intégrales*, and *Sur les quadratures*, afterwards published in *Liouville's Journal.*

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SCIENTIFIC LITERATURE.

Les Oscillations Électriques. H. POINCARÉ.
(CONCLUDED.)

Propagation of Electrical Oscillations Through Air.—The velocity of propagation of electromagnetic induction through dielectrics offered the first experimental test of superiority of the Faraday-Maxwell theory over the older theories. According to these that velocity should be infinite; according to the Faraday-Maxwell view of electromagnetic phenomena it should be the same as that of light. Poincaré reviews carefully all the experimental evidences bearing upon this point. Hertz's experiments in Carlsruhe are first discussed and his early failures in arriving at a satisfactory result are pointed out. Two methods employed in these measurements by Hertz at Carlsruhe and at Bonn are described briefly. One of these consisted in measuring by means of a resonator the difference of phase between two waves sent forth by the same oscillator, one wave along a conducting wire and the other through the dielectric in the vicinity of the wire. The other method consisted in measuring what Hertz considered the wave length of stationary electric waves in air formed by the interference between the direct waves sent forth by an oscillator and the waves reflected by a large flat mirror consisting of a metal sheet 2 meters wide and 4 meters high. In all these experiments the velocity of propagation along the wire seemed to come out considerably different from and generally less than that in air. But the methods were open to several criticisms. In the first place, the hall in which these experiments were carried out was too small for the wave lengths employed; secondly, the influence of the waves reflected from the walls was entirely neglected; thirdly, the dimensions of the reflecting mirror were not large enough in comparison to the wave length to prevent errors of observation due to the misleading influence of diffraction phenomena. All these objections were in a measure overcome in the earliest experiments of Sarasin and de la Rive (C. R. t. CX. p. 72). In these experiments the methods of Hertz were employed, but they were performed in a large hall, with a large